

# 3

# Vectors and Projectiles

## 3-1 Vectors and Scalars

*Vocabulary*

**Vector:** A quantity with magnitude (size) and direction.

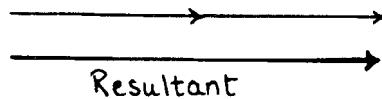
Some examples of vectors are displacement, velocity, acceleration, and force.

*Vocabulary*

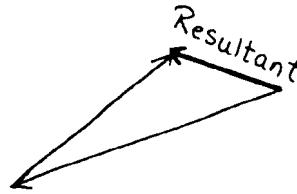
**Scalar:** A quantity with magnitude only.

Some examples of scalars are distance, speed, mass, time, and volume.

Vectors are represented by arrows. They can be added by placing the arrows head to tail. The arrow that extends from the tail of the first vector to the head of the last vector is called the **resultant**. It indicates both the magnitude and direction of the vector sum.

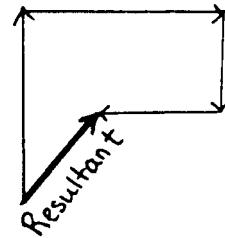


Remember, vectors don't always have to be in a straight line but may be oriented at angles to each other, such as shown.

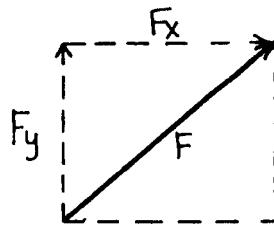


Resultant vectors can be determined by a number of different methods. Here you will solve vector addition exercises both **graphically** and with **vector components**.

**Graphical addition of vectors:** Using a ruler, draw all vectors to scale and connect them head to tail. The resultant is the vector that connects the tail of the first vector with the head of the last. (*Hint:* Using graph paper makes this method even easier!)



**Vector Components:** Because a vector has both magnitude and direction, you can separate it into horizontal (or  $x$ ) and vertical (or  $y$ ) components. To do this, draw a rectangle with horizontal and vertical sides and a diagonal equal to the vector. Draw arrow heads on one horizontal and one vertical side to make the original vector the resultant of the horizontal and vertical components.



After you have drawn the components, you can find their lengths by using simple trigonometry. If you are not familiar with trigonometry or need a quick refresher, refer to Appendix A.

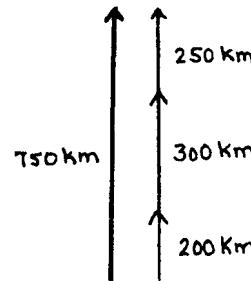
## Solved Examples

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**Example 1:** Every March, the swallows return to San Juan Capistrano, California after their winter in the south. If the swallows fly due north and cover 200 km on the first day, 300 km on the second day, and 250 km on the third day, draw a vector diagram of their trip and find their total displacement for the three-day journey.

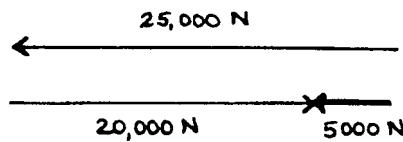
**Solution:** Because the swallows continue to fly in the same direction throughout the entire trip, these vectors simply add together. This can be shown by placing the displacement vectors head to tail.

$$200 \text{ km} + 300 \text{ km} + 250 \text{ km} = 750 \text{ km north}$$



**Example 2:** In the record books, there are men who claim that they have such strong teeth that they can even use them to move cars, trains, and helicopters. Joe Ponder of Love Valley, North Carolina is one such man. Suppose a car pulling forward with a force of 20,000 N was pulled back by a rope that Joe held in his teeth. Joe pulled the car with a force of 25,000 N. Draw a vector diagram of the situation and find the resultant force.

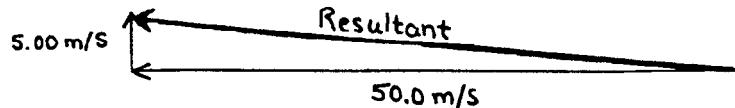
**Solution:** In this exercise, the vectors are pointing in opposite directions, so the situation would look like this.



$$25,000 \text{ N} - 20,000 \text{ N} = 5000 \text{ N in the direction Joe is pulling. Strong teeth!}$$

**Example 3:** On August 7, 2007, Barry Bonds of the San Francisco Giants passed Hank Aaron to become Major League Baseball's career home run leader, by hitting his 756th homerun. If Bonds hits a baseball due west with a speed of 50.0 m/s, and the ball encounters a wind that blows it north at 5.00 m/s, what is the resultant velocity of the baseball? (Read more about Barry Bonds' record setting homerun at <http://www.barrybonds.com>)

**Solution:** Begin by drawing a vector diagram of the situation.



Solve using the Pythagorean theorem:

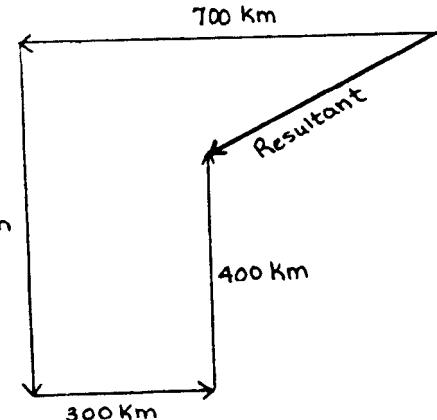
$$\begin{aligned} a^2 + b^2 &= c^2 \\ (50.0 \text{ m/s})^2 + (5.00 \text{ m/s})^2 &= c^2 \\ c = \sqrt{2500 \text{ m}^2/\text{s}^2 + 25.0 \text{ m}^2/\text{s}^2} &= 50.2 \text{ m/s toward the northwest} \end{aligned}$$

For those of you who understand trigonometry, you can find the exact angle at which the ball travels by saying:

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{5.00 \text{ m/s}}{50.0 \text{ m/s}} = 0.100 \quad \text{so } \tan^{-1} 0.100 = 5.71^\circ$$

**Example 4:** The Maton family begins a vacation trip by driving 700 km west. Then the family drives 600 km south, 300 km east, and 400 km north. Where will the Matons end up in relation to their starting point? Solve graphically.

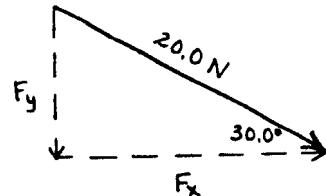
**Solution:** First, draw the appropriate diagram to scale using a relationship such as 1 cm = 1 km, and you will see a space remaining between where the Matons began their trip and where they ended. Because you are solving this exercise graphically, measure with a ruler the length of the remaining space and convert your measurement back into km. This is the resultant displacement. (*Hint:* You may find it easier to use graph paper for your drawing so that you can have 1 km equal to a certain number of squares.)



Answer is 450 km, as measured with a ruler.

**Example 5:** Ralph is mowing the back yard with a push mower that he pushes downward with a force of 20.0 N at an angle of 30.0° to the horizontal. What are the horizontal and vertical components of the force exerted by Ralph?

**Solution:** Begin solving by drawing a diagram of the situation, labeling the horizontal and vertical components of the force.



**Horizontal component:** The hypotenuse in this exercise is the 20.0-N force. The horizontal component is the one going in the  $x$  direction. This is the side adjacent to the 30.0° angle so you use the equation for the cosine of an angle.

$$\cos \theta = \frac{F_x}{F} \quad F_x = F \cos \theta = (20.0 \text{ N}) \cos 30.0^\circ = 17.3 \text{ N}$$

**Vertical component:** Again, the 20.0-N force is the hypotenuse of the triangle. The vertical component is the one going in the  $y$  direction. This is the side opposite the 30.0° angle so you use the equation for the sine of an angle.

$$\sin \theta = \frac{F_y}{F} \quad F_y = F \sin \theta = (20.0 \text{ N}) \sin 30.0^\circ = 10.0 \text{ N}$$

## Practice Exercises

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**Exercise 1:** Some Antarctic explorers heading due south toward the pole travel 50. km during the first day. A sudden snowstorm slows their progress and they move only 30. km on the second day. With plenty of rest, they travel the final 65 km the last day and reach the pole. What was the explorers' displacement?

Answer: \_\_\_\_\_

**Exercise 2:** Erica and Tory are out fishing on the lake on a hot summer day when they both decide to go for a swim. Erica dives off the front of the boat with a force of 45 N, while Tory dives off the back with a force of 60. N. a) Draw a vector diagram of the situation. b) Find the resultant force on the boat.

Answer: b. \_\_\_\_\_

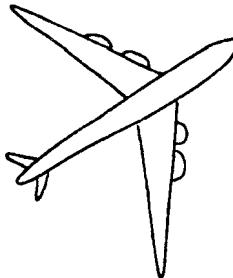
**Exercise 3:** Young thoroughbreds are sometimes reluctant to enter the starting gate for their first race. Astro Turf is one such horse, and it takes two strong men to get him set for the race. Derek pulls Astro Turf's bridle from the front with a force of 200. N and Dan pushes him from behind with a force of 150. N, while the horse pushes back against the ground with a force of 300. N. a) Draw a vector diagram of the situation. b) What is the resultant force on Astro Turf?

Answer: b. \_\_\_\_\_

**Exercise 4:** Shareen finds that when she drives her motorboat upstream she can travel with a speed of only 8 m/s, while she moves with a speed of 12 m/s when she heads downstream. What is the current of the river on which Shareen is traveling?

Answer: \_\_\_\_\_

- Exercise 5:** Rochelle is flying to New York for her big Broadway debut. If the plane heads out of Los Angeles with a velocity of 220. m/s in a northeast direction, relative to the ground, and encounters a wind blowing head-on at 45 m/s, what is the resultant velocity of the plane, relative to the ground?



Answer: \_\_\_\_\_

- Exercise 6:** While Dexter is on a camping trip with his boy scout troop, the scout leader hands each boy a compass and map. The directions on Dexter's map read as follows: "Walk 500.0 m north, 200.0 m east, 300.0 m south, and 400.0 m west." If he follows the map, what is Dexter's displacement? Solve graphically.

Answer: \_\_\_\_\_

- Exercise 7:** Amit flies due east from San Francisco to Washington, D.C., a displacement of 5600. km. He then flies from Washington to Boston, a displacement of 900. km at an angle of  $55.0^\circ$  east of north. What is Amit's total displacement?

Answer: \_\_\_\_\_

**Exercise 8:** Marcie shovels snow after a storm by exerting a force of 30.0 N on her shovel at an angle of  $60.0^\circ$  to the vertical. What are the horizontal and vertical components of the force exerted by Marcie?

Answer: \_\_\_\_\_

Answer: \_\_\_\_\_

**Exercise 9:** Ivan pulls a sled loaded with logs to his cabin in the woods. If Ivan pulls with a force of 800. N in a direction  $20.0^\circ$  above the horizontal, what are the horizontal and vertical components of the force exerted by Ivan?



Answer: \_\_\_\_\_

Answer: \_\_\_\_\_